AN ANALYSIS TO FACILITATE THE
INTERPRETATION OF BOUNDARY
LAYER MEASUREMENTS

W. C. Zierke and S. Deutsch

Technical Memorandum
File No. TM 85-85
25 May 1985
NASA Grant NSG-3264

Copy No. 17

The Pennsylvania State University
Intercollege Research Programs and Facilities
APPLIED RESEARCH LABORATORY
Post Office Box 30
State College, PA 16804

NATIONAL AERONAUTICS & SPACE ADMINISTRATION

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER TM 85-85	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) AN ANALYSIS TO FACILITATE THE INTERPRETATION OF BOUNDARY LAYER MEASUREMENTS		5. TYPE OF REPORT & PERIOD COVERED Technical Memorandum 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) W. C. Zierke and S. Deutsch		8. CONTRACT OR GRANT NUMBER(s) NSG-3264
9. PERFORMING ORGANIZATION NAME AND ADDRESS Applied Research Laboratory Post Office Box 30 State College, PA 16804		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS NASA Lewis Research Center 21000 Brookpark Road Cleveland, OH 44135		12. REPORT DATE 25 May 1985 13. NUMBER OF PAGES 38
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release. Distribution unlimited.
Per NASA -- 2 July 1985.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A computer aided boundary layer analysis, which draws on available theoretical and empirical data, is presented. The analysis allows boundary layer data to be extracted from measurements influenced by a normal pressure gradient, comparison of boundary layer measurements against standard laminar and turbulent profiles, and evaluation of important boundary layer parameters. This technique is illustrated by example.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Subject: An Analysis to Facilitate the Interpretation of Boundary Layer Measurements

Abstract: A computer aided boundary layer analysis, which draws on available theoretical and empirical data, is presented. The analysis allows boundary layer data to be extracted from measurements influenced by a normal pressure gradient, comparison of boundary layer measurements against standard laminar and turbulent profiles, and evaluation of important boundary layer parameters. This technique is illustrated by example.

Acknowledgments: This work was supported by NASA Grant NSG-3264 with Mr. Nelson Sanger acting as grant manager.

25 May 1985
WCZ:SD:lhz

Nomenclature

C	Law of the wall constant (= 5.0)
C_f	Skin friction coefficient
E	Error between the data and the wall-wake equation
f	Similarity function
G	Clauser's shape factor
H_{12}	First shape factor
H_{32}	Second shape factor
ℓ	Reference length
N	Number of data points
N_{inv}	Number of data points in the inviscid region
N_{max}	Maximum number of data points that could possibly be in the inviscid region
p	Static pressure
p_e	Static pressure at the boundary layer edge
Re	Reference Reynolds number
Re_θ	Momentum thickness Reynolds number
u	Streamwise velocity
$u_{b\ell}$	Boundary layer velocity
u_{inv}	Inviscid velocity
u_{meas}	Measured composite velocity
u^+	Dimensionless velocity in the inner boundary layer
u_τ	Shear or friction velocity

Nomenclature [continuation]

U_e	Velocity at the boundary layer edge
U_{ref}	Reference velocity
v	Normal velocity
W	Coles' universal wake function
x	Streamwise coordinate
y	Coordinate normal to the blade surface
y^+	Dimensionless coordinate normal to the blade surface in inner boundary layer variables
β	Falkner-Skan streamwise pressure gradient parameter
β_c	Clauser's equilibrium parameter
δ	Boundary layer thickness
δ_3	Energy thickness
δ^*	Displacement thickness
Δ	Defect thickness
η	Dimensionless normal similarity variable
θ	Momentum thickness
κ	Von Karman's mixing length parameter (= 0.41)
ν	Kinematic viscosity
ξ	Dimensionless streamwise similarity variable
Π	Coles' wake parameter
ρ	Fluid density
τ_w	Wall shear stress
ψ	Streamfunction

25 May 1985
WCZ:SD:lhz

Nomenclature [continuation]

Subscripts

i	The i^{th} data point
wall	At the wall
ξ	Derivative with respect to ξ

Superscripts

$'$	Derivative with respect to η

Introduction

Over the past several years, detailed boundary layer velocity profiles have been made using laser Doppler velocimetry in the periodic two-dimensional flow over a double circular arc compressor blade in cascade. The initial incidence angle was 5° so that laminar, transitional, turbulent and separated velocity profiles were all encountered. Two typical profiles, one on the suction surface and one on the pressure surface, are shown in Figures 1 and 2. The effect of a strong normal pressure gradient is apparent and even the most fundamental properties of the boundary layers are masked. Moreover, experience has shown that even without the complication of a strong normal pressure gradient, the state of a boundary layer (e.g., laminar or turbulent) is often not self evident and the method for calculating the properties of the layer are often not obvious.

In the following, a method of boundary layer analysis, which draws on available theoretical and empirical formulations, is presented. The analysis details methods of reconstructing boundary layer profiles from data influenced by a normal pressure gradient, of comparing data against standard laminar and turbulent profiles (with or without streamwise pressure gradients), and of generating important boundary layer parameters (e.g., thicknesses or shear velocity). The analysis is complete for laminar, transitional, and turbulent boundary layers. Extension of the techniques to separated profiles and wakes is underway.

Normal Pressure Gradient

Surface curvature, blowing, or suction may induce significant streamline curvature in a flow. Streamline curvature in turn causes a normal pressure gradient which results in a cross streamline gradient in the inviscid velocity profile. As in Figures 1 and 2 then, the freestream velocity does not reach a constant value (the edge velocity, U_e). Boundary layer analyses, however, assume that the location of the boundary layer edge is known and that a constant freestream velocity exists outside the boundary layer. To use or formulate a boundary layer analysis, a procedure must be used to account for the effect of the normal pressure gradient.

One method which accounts for these effects was outlined by Mellor and Wood [1971] and Ball, Reid and Schmidt [1983]. The method assumes that the measured velocity profiles represent composite profiles. This implies that each of the profiles has a region where the viscous effects predominate, a region in which viscous effects are negligible, and an intermediate region in which the viscid-inviscid results match. Mathematically, the measured composite profile is the sum of a boundary layer profile and an inviscid profile, less what appears in both. The last quantity is commonly called the edge velocity, U_e. That is

$$u_{meas} = u_{b\ell} + u_{inv} - U_e \quad .$$

Clearly, both the boundary layer velocity $u_{b\ell}$ and the measured velocity u_{meas} must go to zero at the wall, so that

$$U_e = \left(u_{inv}\right)_{wall}$$

and the scheme reduces to finding the value of the edge velocity at the wall.

The validity of applying this procedure to velocity data is difficult to establish. However, it does allow for a consistent method to analyze measured velocity profiles provided that the inviscid region can be properly identified from the data. There does not appear to be a rigorous way to do this, so that as in the prior reported study (Deutsch and Zierke [1984]), a consistent method which produces plausible results is adopted.

A least squares technique is used to fit a polynomial to the inviscid velocity profile. Since the inviscid profiles may have significant curvature, the option of fitting a linear, quadratic, or cubic polynomial is provided in the analysis. The difficulty lies in choosing the data points to be included in the least squares analysis. To minimize this problem, the number of data points used in the polynomial fit was varied. First, a maximum number of points, N_{max}, which could possibly be within the inviscid region was determined. For $\partial p/\partial y > 0$ (see Figure 1), N_{max} is taken to be the number of points between the point furthest from the wall and the point of maximum velocity. For $\partial p/\partial y < 0$ (see Figure 2), the data point at which the profile slope changes by at least 50% is used instead of the point of maximum velocity. For the data in Figure 1, N_{max} was determined to be 19, while N_{max} was found to be 24 for the data of Figure 2. Many applications have shown that for the region of $0.55\ N_{max} < N_{inv} < 0.95\ N_{max}$, the value of U_e is relatively independent of the number of points in the fit. This is clearly shown for the data of Figures 1 and 2 in Table 1. Each polynomial was extrapolated to the wall to obtain U_e and a mean and a standard deviation of all values of U_e were determined. The degree of the polynomial was chosen to minimize the standard deviation of U_e. These standard deviations have been observed to be quite small (~ 0.5%). A $u_{b\ell}$ profile was calculated using the mean value of U_e and a polynomial fit of u_{inv}. For u_{inv}, $N_{inv} = 0.75\ N_{max}$ was chosen as the number of data points in the fit as the U_e found in this way was consistently close to the average U_e. A smoothed spline fit of the boundary layer velocity profile was used to calculate the boundary layer thickness, δ. δ is taken as the position at which

$$u_{b\ell} = 0.99\ U_e\ .$$

In Figures 3 and 4, the reconstructed boundary layer profiles (triangles) corresponding to Figures 1 and 2 are shown. As might be anticipated, the effect of the normal pressure gradient does not penetrate far into the boundary layer.

Deutsch and Zierke [1984] discuss the plausibility of this approach by considering a comparison of the shape of the resulting turbulence intensity versus normalized distance (y/δ) from the wall plots against classical measurements. They conclude that the technique does indeed give reasonable results. It should be noted that an alternate method of accounting for the effects of the normal pressure gradient using the equations for the boundary layer parameters was developed by Kiock [1983]. That technique was found here, however, to be too difficult to employ.

Preliminary Boundary Layer Analysis

Comparisons between boundary layers are often made in terms of simple integral thicknesses or their ratios (shape factors).

In order to calculate the boundary layer integral parameters, the velocity data points must be fit with a mathematical curve. A parametric cubic spline was used for the curve fit. This curve parametrically develops the $u_{b\ell}$ and y ordered pairs as independent functions of the overall arc length of the curve. Even this curve fit tends to oscillate when there is a cusp in the profile. Therefore, a smoothing routine was added to the spline fit. Near the wall, a parametric cubic spline was fit between the data point nearest to the wall and the zero velocity point that would occur at the wall. For boundary layer profiles not well resolved in the near wall region, this latter fit represents the largest potential error in the calculation of the integral parameters. Shown in Figures 5 and 6 are the calculated spline fits to the boundary layer profiles of Figures 3 and 4.

Boundary Layer Integral Parameters

The most common of the parameters which characterize the boundary layer are integral thicknesses based on the conservation of mass, momentum and energy. In effect, the boundary layer acts to displace the streamlines in the flow outside the boundary layer away from the wall. A displacement thickness, δ^*, can be defined as the distance by which the solid surface would have to be displaced to maintain the same mass flowrate in a hypothetical inviscid flow. For steady, incompressible flow, δ^*, can be defined as

$$\delta^* = \int_0^\infty \left(1 - \frac{u_{b\ell}}{U_e}\right)dy \quad .$$

δ^* is probably the most fundamental integral thickness and is commonly used as a normalizing factor in presenting data since the alternative, the boundary layer thickness, is difficult to measure. A momentum thickness, θ, can be determined from the steady, incompressible momentum equation as

$$\theta = \int_0^\infty \frac{u_{b\ell}}{U_e}\left(1 - \frac{u_{b\ell}}{U_e}\right)dy \quad .$$

The momentum thickness represents the momentum loss due to the presence of the boundary layer and is proportional to the drag when no streamwise pressure gradient exists. The momentum thickness is used in many empirical correlations. Finally, a steady, incompressible energy thickness can be defined as

$$\delta_3 = \int_0^\infty \frac{u_{b\ell}}{U_e} \left(1 - \frac{u_{b\ell}^2}{U_e^2}\right) dy \quad .$$

Reynolds numbers can be formed based on all three of these integral thicknesses using the boundary layer edge velocity. They are especially useful in describing the boundary layers just before, during, and just after transition.

Some parameters describe the shape of the velocity profile. The first and second shape factors of the velocity profile are defined as

$$H_{12} = \frac{\delta^*}{\theta}$$

and

$$H_{32} = \frac{\delta_3}{\theta} \quad .$$

Note that the definitions require that both shape factors be greater than unity. For laminar boundary layers, the first shape factor lies between 3.5 and 2.3. Transition brings about a considerable drop in H_{12} giving a turbulent boundary layer in which H_{12} lies between 1.3 and 2.2. Laminar separation takes place at a value of H_{12} near 3.5 while turbulent separation takes place at a value of H_{12} near 2.2.

In the current study, the integral parameters and shape factors were found by integrating the spline fit using a trapezoidal rule with very fine spacing. Values of these parameters and the associated Reynolds numbers are given in Table 2 for each of the profiles shown in Figures 5 and 6. Based on the work of Purtell, Klebanoff and Buckley [1981] and Murlis, Tsai. and Bradshaw [1982], one would anticipate that the Re_θ for the profile of Figure 6 is too low to support turbulence.

Similarity Solutions for Laminar Boundary Layers

For a constant streamwise pressure gradient, laminar boundary layers can be plotted in such a way as to collapse them all to the same curve. Such laminar boundary layers are said to be self-similar and solutions to the boundary layer momentum equation may be solved using a similarity solution. A similarity solution reduces the number of variables in the equation by using a coordinate transformation. The boundary layer momentum equation for steady flow can be written as

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{dp_e}{dx} + \nu \frac{\partial^2 u}{\partial y^2} \quad .$$

Several different coordinate transformations have been used for similarity solutions. One such transformation is the Levy-Lees transformation (see Cebeci and Smith [1974]) which is given as

$$\xi = \int_0^{x/\ell} \left(\frac{U_e}{U_{ref}}\right) d\left(\frac{x}{\ell}\right)$$

and

$$\eta = \frac{1}{\sqrt{2\xi}} \left(\frac{U_e}{U_{ref}}\right)\left(\frac{y}{\ell}\right) \sqrt{Re}$$

where

$$Re = \frac{\ell U_{ref}}{\nu} \quad .$$

Again, these transformation variables are for an incompressible flow with a constant value of viscosity. The variables U_{ref} and ℓ refer to a reference velocity and a reference length, respectively. A similarity function, f, can then be defined from the streamfunction, ψ, as

$$\psi(x,y) = \sqrt{2\xi}\, f(\xi,\eta)$$

where

$$u = \frac{\partial \psi}{\partial y}$$

and

$$v = -\frac{\partial \psi}{\partial x} \quad .$$

The boundary layer momentum equation for steady, incompressible flow can now be transformed into the following differential equation

$$f''' + ff'' + \beta(1 - f'^2) = 2\xi(f' f'_\xi - f_\xi f'')$$

where

$$\beta = \frac{2\xi}{U_e} \frac{dU_e}{d\xi} \quad .$$

β is a streamwise pressure gradient parameter and must be determined from the measured streamwise static pressure distribution. These measurements are also required to determine the values of ξ. When f has a prim. ıɔr a superscript, it refers to a derivative with respect to η. A derivative with respect to ξ is noted by an ξ for a subscript.

A similarity solution which is quite useful was developed by Falkner and Skan [1931]. Their solution is for laminar boundary layers flowing over wedges where the streamwise pressure gradients are constant. In terms of the Levy-Lees coordinates, the Falkner-Skan solution involves no changes in the ξ-direction. Thus all derivatives with respect to ξ are zero and the boundary layer momentum equation for steady, incompressible flow is reduced to the following ordinary differential equation

$$f''' + ff'' + \beta(1 - f'^2) = 0 \quad .$$

The solution of this equation is found in the present boundary layer analysis from the solution scheme of Hoffman [1983]. The Falkner-Skan solution represents a very good approximation of the laminar boundary layer solution in two cases: first, when the streamwise pressure gradient changes only slowly in the streamwise direction, and second, near the leading edge of the boundary layer surface (since ξ is very small and the Levy-Lees transformed equation is approximated quite well by the Falkner-Skan equation). In the equation above when β is equal to zero, the Falkner-Skan solution is reduced to the original similarity solution of Blasius [1908] for laminar boundary layers with no streamwise pressure gradient. Separation of the laminar boundary layer will occur for a β with a value of -0.199.

The profile of Figure 6 is shown compared against a Falkner-Skan profile at an equivalent β in Figure 7. Although the streamwise pressure gradient is not constant here, the comparison is quite good. The profile of Figure 6 (via Figures 2 and 4) is laminar. Integral parameters and skin friction values can be computed from the Falkner-Skan solution. These are presented in Table 2; where appropriate they are compared to the values calculated from the spline fit.

Wall-Wake Velocity Profile for Turbulent Boundary Layers

Turbulent boundary layers are commonly divided into different regions. The innermost region is dominated by viscous shear and is self-similar for all turbulent boundary layers. This region is called the viscous sublayer and is described by

$$\frac{u}{u_\tau} = \frac{y u_\tau}{\nu}$$

or

$$u^+ = y^+$$

where u^+ and y^+ are called inner variables. u_τ is called the shear or friction velocity. Outside of the sublayer but still very close to the wall, the velocity is logarithmic with distance as

$$u^+ = \frac{1}{\kappa} \ln(y^+) + C \quad .$$

The viscous sublayer and logarithmic region overlap in a region called the buffer layer. Collectively the sublayer, buffer layer and logarithmic layer are called the "law of the wall." Streamwise pressure gradients have essentially no effect on this region. Outside of this wall region, the streamwise pressure gradients are important and the velocity profile exhibits a wake-like form. Coles [1956] developed an equation for the wake region called the "law of the wake." His composite equation included a wake-like function added to the logarithmic equation. This wall-wake equation can be written as

$$u^+ = \frac{1}{\kappa} \ln(y^+) + C + \frac{\Pi}{\kappa} W\left(\frac{y}{\delta}\right)$$

where

$$W\left(\frac{y}{\delta}\right) = 2 \sin^2\left(\frac{\pi y}{2\delta}\right) = 1 - \cos\left(\frac{\pi y}{\delta}\right) \quad .$$

W is called Coles' universal wake function and is normalized to be zero at the wall and has a value of $W = 2.0$ at $y = \delta$. Coles' wake parameter Π brings in the effect of the streamwise pressure gradient and has a value of approximately 0.5 for a zero pressure gradient flow. Coles and Hirst [1968] later examined the data from a number of turbulent boundary layer experiments and determined that the von Karman mixing length parameter, κ, should be 0.41 and the law of the wall constant, C, should be 5.0.

The analysis of turbulent boundary layers should include a fit of the data to the wall-wake equation. A least squares fit of the data to the equation was chosen with u_τ and Π as the variables to be determined and δ considered to be a known quantity. The error between each data point and the wall-wake equation is

$$E_i = \frac{u_\tau}{\kappa} \ln\left(\frac{y_i u_\tau}{\nu}\right) + u_\tau C + \frac{u_\tau \Pi}{\kappa} \left[1 - \cos\left(\frac{\pi y_i}{\delta}\right)\right] - u_i \quad .$$

Now the minimum error squared of all the data points must be found with respect to both u_τ and Π. Taking the partial derivative of the error squared for each data point with respect to u_τ and Π yields

$$\frac{\partial E_i^2}{\partial u_\tau} = 2E_i \left\{\frac{1}{\kappa} \ln\left(\frac{y_i u_\tau}{\nu}\right) + \frac{1}{\kappa} + C + \frac{\Pi}{\kappa} \left[1 - \cos\left(\frac{\pi y_i}{\delta}\right)\right]\right\}$$

and

$$\frac{\partial E_i^2}{\partial \Pi} = 2E_i \left\{\frac{u_\tau}{\kappa} \left[1 - \cos\left(\frac{\pi y_i}{\delta}\right)\right]\right\} \quad .$$

Summing for all the data points and setting the two expressions equal to zero gives two equations

$$\sum_{i=1}^{N} \frac{\partial E_i^2}{\partial u_\tau} = 0$$

and

$$\sum_{i=1}^{N} \frac{\partial E_i^2}{\partial \Pi} = 0$$

which can be solved simultaneously for u_τ and Π to give the minimum squared error. A secant method is used to solve the simultaneous non-linear equations. A similar method was presented by Sun and Childs [1976] for compressible turbulent boundary layers using the velocity transformation of van Driest [1951].

In order to use the fit of the wall-wake equation, the range of data points to be used in the fit must be considered. White [1974] states that the logarithmic region does not hold for $y^+ < 35$ (corresponding roughly to $y/\delta < 0.02$). If the wake component is large, Coles and Hirst [1968] state that the region being fit should not include $y/\delta > 0.9$. They suggest that this number should be reduced to 0.75 for a zero streamwise pressure gradient and 0.6 for a vanishing wake component. Here the region $0.05 < y/\delta < 0.75$ was used: sensitivity to the specific range of data points was not large.

The data of Figures 5 (via Figures 1 and 3) is shown in Figure 8 in inner variables. Parameters determined from the wall-wake fit are given in Table 2.

Streamwise pressure gradients can have a strong effect on the outer region of a turbulent boundary layer. Typical profiles at an Re_θ of 5,000 for values of Π of - 0.5, 0.5, 2.0 and 5.0 are shown in Figure 9(a) for illustration. We should note that the value of Re_θ controls the size of the lograithmic region. Shown in Figure 9(b) are the computer profiles for Re_θ of 500, 1,000, 5,000 and 10,000 and a Π of 0.5. Note the difficulty in discerning a logarithmic region for Re_θ of 500.

Most analyses, short of direct computation, consider boundary layers in which the pressure gradient is constant, so that the profiles are self-similar with downstream distance -- so called equilibrium layers. Clauser [1954,1955] conceived of a parameter as

$$\beta_c = \frac{\delta^*}{\tau_w} \frac{dp_e}{dx} \quad .$$

to characterize equilibrium. Here τ_w is the wall shear stress. A turbulent boundary layer with a constant β_c has outer region similarity and is called an equilibrium turbulent boundary layer. All of the gross properties of that boundary layer can be described with a single parameter. Clauser [1954,1955] chose the parameter G, where

$$G = \frac{1}{\Delta} \int_0^\infty \left(\frac{U_e - u_{b\ell}}{u_\tau}\right)^2 dy$$

and

$$\Delta = \int_0^\infty \left(\frac{U_e - u_{b\ell}}{u_\tau}\right) dy \quad .$$

G is called Clauser's shape factor and Δ is termed the defect thickness. Values of G and Δ for the profile of Figure 5 are

$$G = 19.47$$

and

$$\Delta = 142.1 \text{ mm} \quad .$$

The cascade profiles are non-equilibrium. In practice, one expects non-equilbrium to be the rule rather than the exception.

Skin Friction Coefficient of a Turbulent Boundary Layer

An empirical expression for the skin friction coefficient of a turbulent boundary layer was developed by Ludweig and Tillman [1949]. The curve-fit expression from their experimental data is

$$C_f \approx 0.246\ Re_\theta^{-0.268}\ 10^{-0.678\ H_{12}}\ .$$

Skin friction coefficient $\left(C_f\right)$, wall shear stress $\left(\tau_w\right)$ and friction velocity $\left(u_\tau\right)$ can be related by

$$C_f = \frac{\tau_w}{\rho U_e^2/2}$$

and

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}\ .$$

The friction velocity calculated from Ludweig and Tillman's expression agrees well with that calculated from the wall-wake fit for all the non-separated turbulent profiles measured.

Transition

Transition represents the region in which a laminar boundary layer becomes turbulent. The length of the transition region depends strongly on, among others, the streamwise pressure gradient and the freestream turbulence intensity. The incomplete transition of the pressure surface profiles is shown in Figure 10. Here the transition is termed incomplete as the profile at 97.8% chord does not contain a logarithmic region; also, Re_θ is below that believed capable of sustaining full turbulence. Note the comparison with the Falkner-Skan profiles; particularly the thickening of the profile in the near wall region.

Conclusions

A computer aided analysis of boundary layer data is described. The analysis allows for

(1) The reconstruction of boundary layer profiles from data influenced by normal pressure gradients (both U_e and δ are determined),

(2) A calculation of boundary layer integral parameters through a smoothed spline fit of the data,

(3) (a) A comparison of the data against a laminar Falkner-Skan profile at equivalent pressure gradient parameter (β) and/or

(b) A curve fit to the law of the wall/wake of Coles (u_τ and Π are determined), and

(4) A calculation of skin friction from Ludweig-Tillman's equation.

The analysis has been successfully used to interpret both cascade boundary layers (air) and the boundary layer on an axisymmetric microbubble body (water). An extension to separated profiles and wake profiles is planned.

References

[1] Ball, C. L., Reid, L. and Schmidt, J. F., "End-Wall Boundary Layer Measurements in a Two Stage Fan," NASA TM 83409 (June 1983).

[2] Blasius, H., "Grenzchichten in Flüssigkeiten mit kleiner Reibung," Z. Angew. Math. Phys. 56(1):1-37 (1908). [English Translation, NACA TM No. 1256.]

[3] Cebeci, T. and Smith, A. M. O., Analysis of Turbulent Boundary Layers, Academic Press (1974).

[4] Clauser, F. H., "Turbulent Boundary Layers in Adverse Pressure Gradients," Journal of the Aeronautical Sciences 21:91-108 (February 1954).

[5] Clauser, F. H., "The Turbulent Boundary Layer," Advances in Applied Mechanics 4:1-51 (1956).

[6] Coles, D. E., "The Law of the Wake in the Turbulent Boundary Layer," Journal of Fluid Mechanics 1:191-226 (1956).

[7] Coles, D. E. and Hirst, E. A., "Computation of Turbulent Boundary Layers," in Proceedings of the AFOSR-IFP-Stanford Conference, Vol. II (August 1968).

[8] Deutsch, S. and Zierke, W. C., "Some Measurements of Boundary Layers on the Suction Surface of Double Circular Arc Blades in Cascades," ARL/PSU TM 84-77, The Applied Research Laboratory, The Pennsylvania State University (3 April 1984).

[9] Falkner, V. M. and Skan, S. W., "Some Approximate Solutions of the Boundary Layer Equations," Phil. Mag. 12(7):865-896 (1931).

[10] Hoffman, G. H., "A Rapid Method for Predicting Suction Distributions to Maintain Attached, Laminar Boundary Layers on Bodies of Revolution," ARL/PSU TM 83-201, Applied Research Laboratory, The Pennsylvania State University (December 1983).

[11] Kiock, R., "Evaluation of Boundary Layer Measurements in Two-Dimensional Compressible Subsonic Flow with a Pressure Gradient across the Boundary Layer," ESA-TT-810, European Space Agency (July 1983).

[12] Ludweig, H. and Tillman, W, "Untersuchungen über die Wandschubspannung in Turbulenten Reibungsschenhten," Ing.-Arch. 17:288-299 (1949). [English Translation, NACA TM 1285.]

[13] Mellor, G. L. and Wood, G. M., "An Axial Compressor End-Wall Boundary Layer Theory," Transactions of the ASME, Journal of Basic Engineering, pp. 300-316 (June 1971).

[14] Murlis, J., Tsai, H. M. and Bradshaw, P., "The Structure of Turbulent Boundary Layers at Low Reynolds Numbers," Journal of Fluid Mechanics 122:13-56 (1982).

[15] Purtell, L. R., Klebanoff, P. S. and Buckley, F. T., "Turbulent Boundary Layers at Low Reynolds Numbers," Physics of Fluids 24:802 (1981).

[16] Sun, C. C. and Childs, M. E., "Wall-Wake Velocity Profile for Compressible Nonadiabatic Flows," AIAA Journal 14(6):820-822 (June 1976).

[17] Van Driest, E. R., "Turbulent Boundary Layer in Compressible Fluids," Journal of the Aeronautical Sciences 18:145-160 and 216 (1951).

[18] White, F. M., Viscous Fluid Flow, McGraw-Hill, Inc. (1974).

Table 1. Variation of U_e and δ with the Number of Parts used in the Polynomial Fit of the Inviscid Region.

Suction Surface Boundary Layer at 53.6% Chord ($\partial p/\partial y > 0$)

N_{inv}	U_e, m/sec	δ, cm
11	33.64	1.130
12	33.74	1.142
13	33.83	1.154
14	34.00	1.177
15	33.84	1.155
16	33.76	1.146
17	33.64	1.132
	33.78 ± 0.13	1.148 ± 0.016

Pressure Surface Boundary Layer at 57.2% Chord ($\partial p/\partial y < 0$)

N_{inv}	U_e, m/sec	δ, cm
14	23.37	0.231
15	23.38	0.233
16	23.39	0.234
17	23.41	0.237
18	23.40	0.235
19	23.40	0.235
20	23.40	0.235
21	23.40	0.235
22	23.40	0.235
	23.39 ± 0.02	0.234 ± 0.002

Table 2. Boundary Layer Parameters.

Parameters	Suction Surface Boundary Layer at 53.6% Chord ($\partial p/\partial y > 0$)		Pressure Surface Boundary Layer at 57.2% Chord ($\partial p/\partial y < 0$)	
	Spline Fit	Fit of Wall-Wake Eq.	Spline Fit	Falkner-Skan Solution
δ^*, (cm)	0.350	0.346	0.083	0.059
θ , (cm)	0.180	0.182	0.029	0.022
δ_3, (cm)	0.288	0.291	0.046	--------
Re_{δ^*}	7881	7795	1295	835
Re_θ	4041	4095	451	310
Re_{δ_3}	6489	6557	213	--------
H_{12}	1.95	1.90	2.87	2.70
H_{32}	1.61	1.60	1.58	--------
u_τ, (m/sec)	0.850[1]	0.823	0.385[1]	0.337
Π	--------	4.496	--------	--------
β_c	--------	7.362	--------	--------
β	--------	--------	--------	- 0.0588

(1) Calculated from the Ludweig-Tillman Equation.

SD:WCZ:lrs
25 May 1985

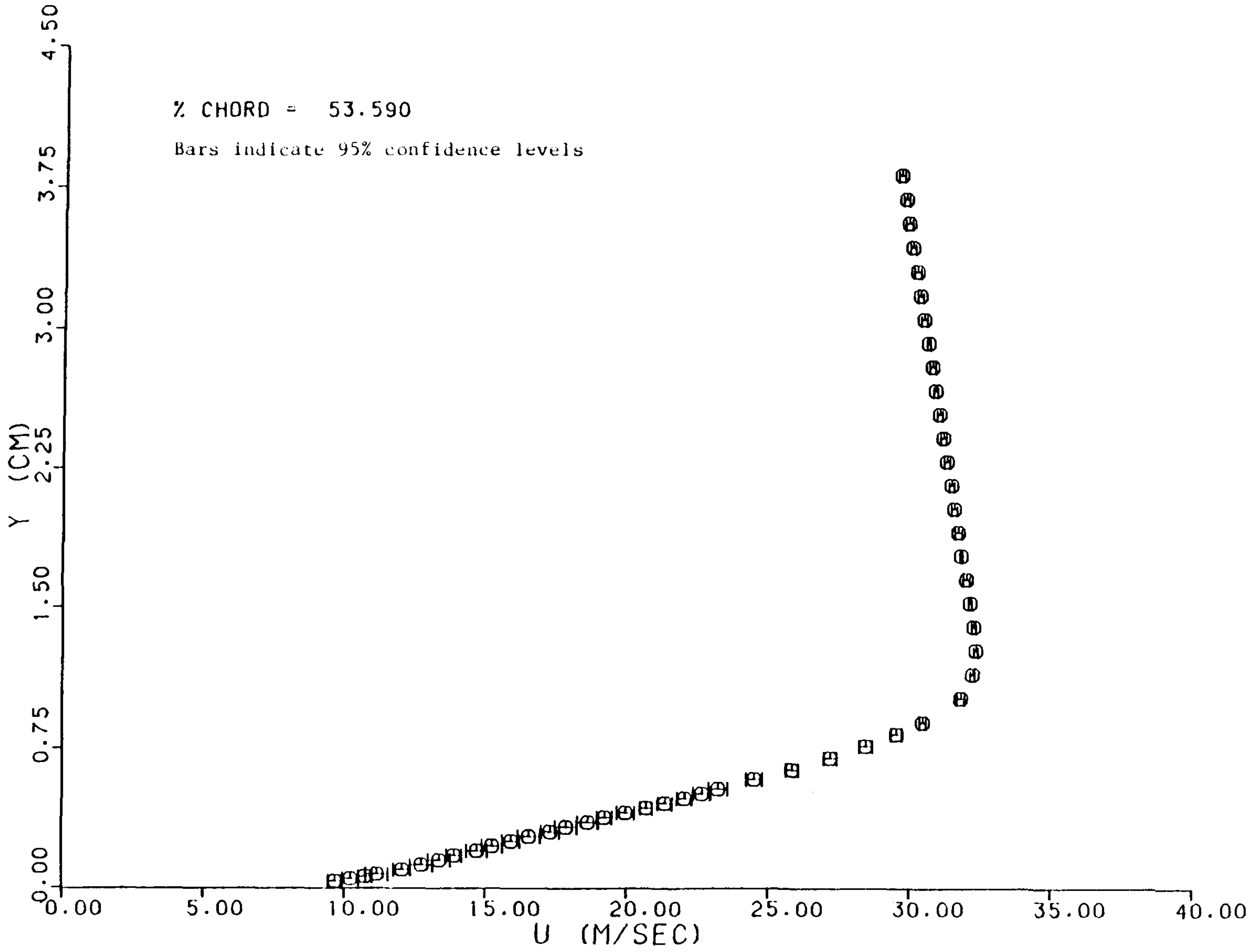


Figure 1. Typical measured velocity profile with $\partial p/\partial y > 0$.

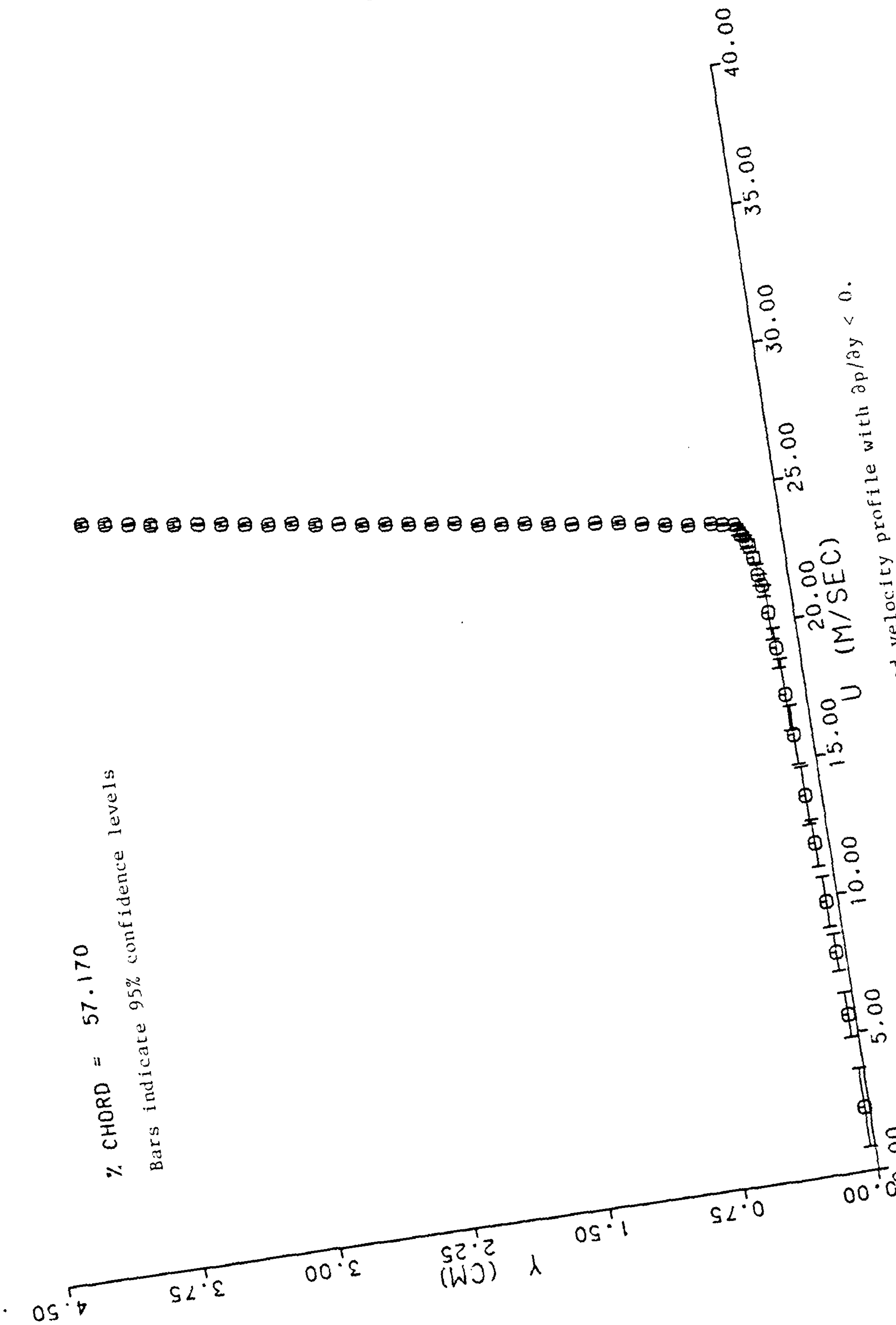


Figure 2. Typical measured velocity profile with $\partial p/\partial y < 0$.

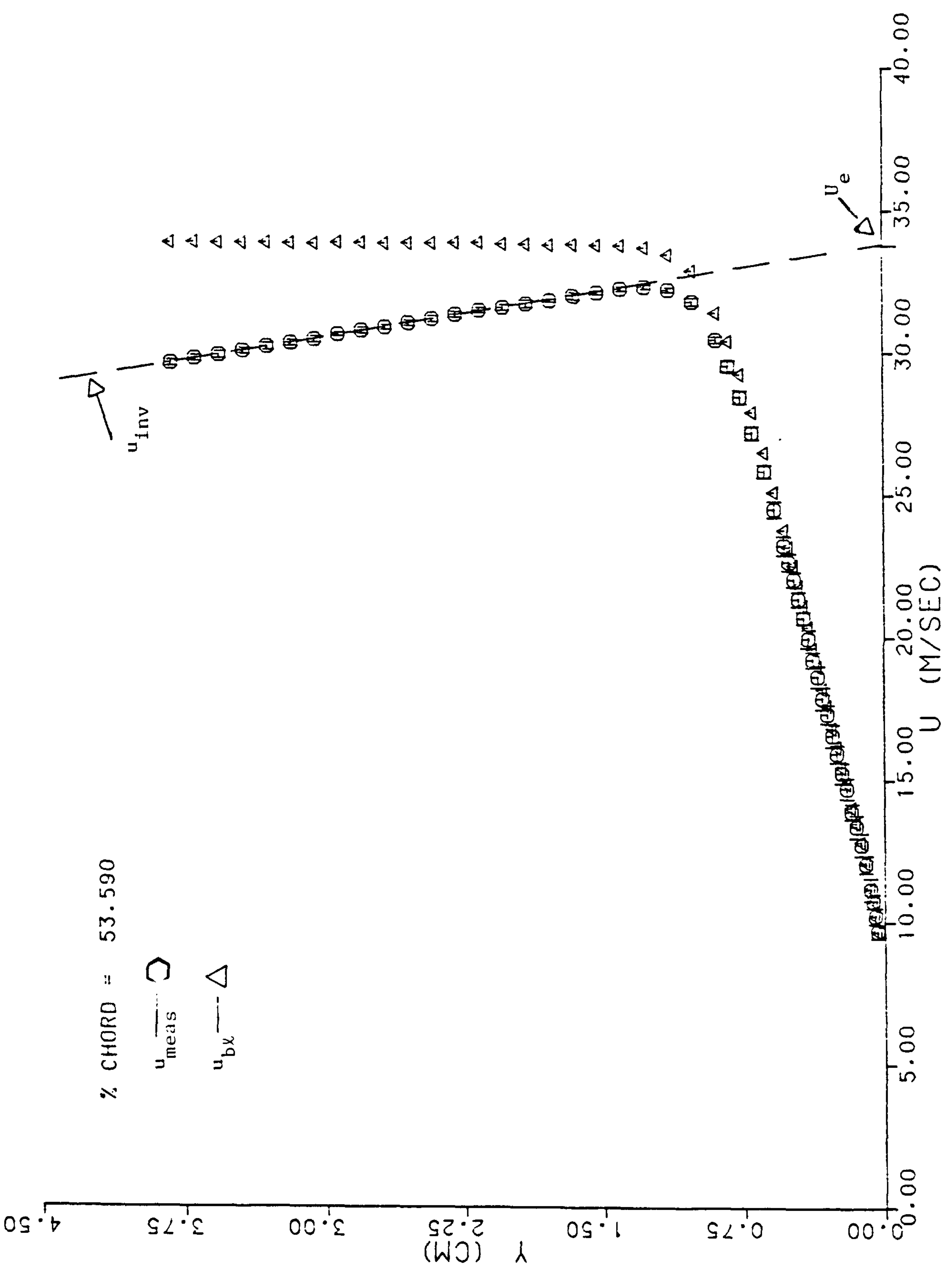


Figure 3. Reconstructed boundary layer velocity profile ($\partial p/\partial y > 0$).

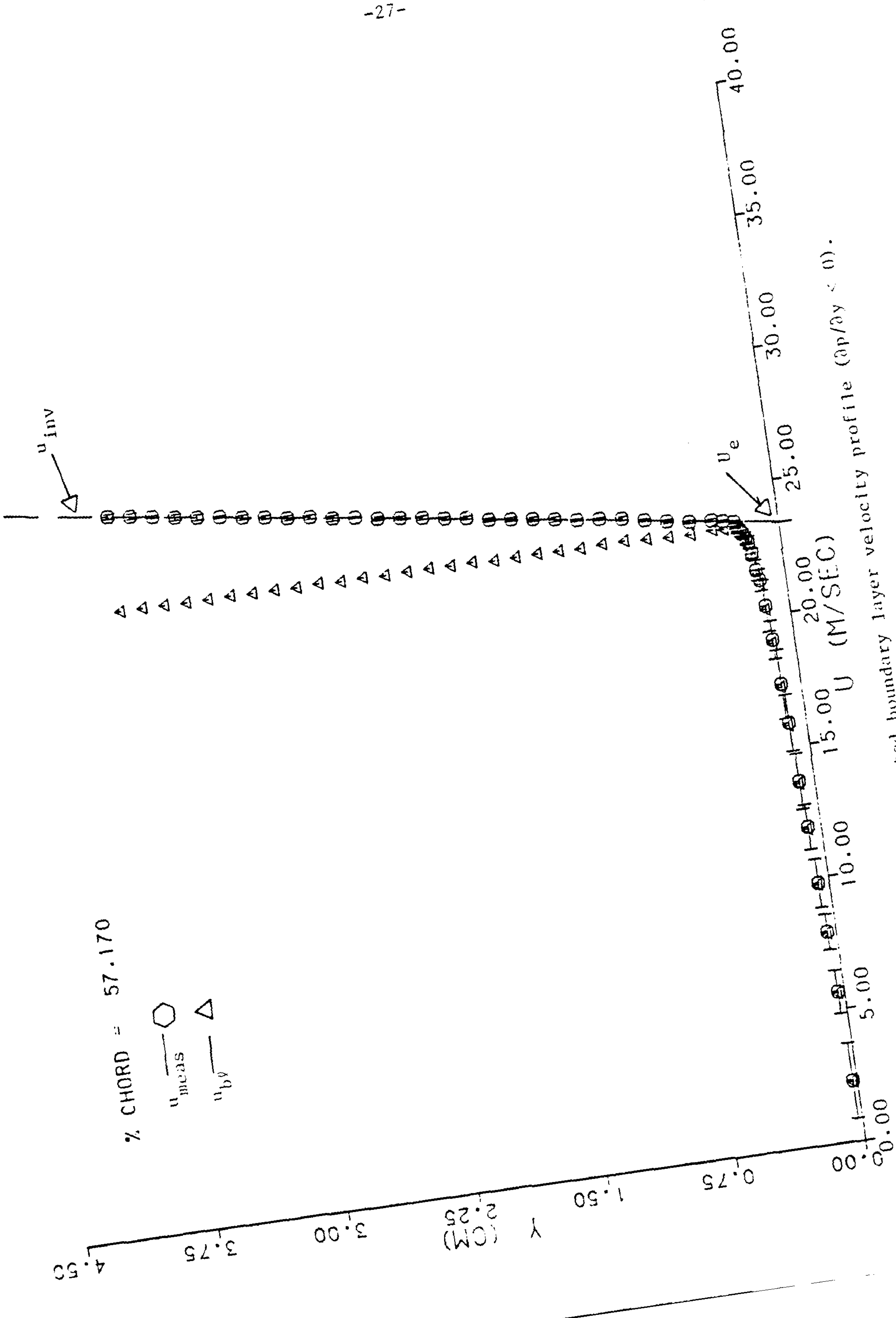


Figure 4. Reconstructed boundary layer velocity profile ($\partial p/\partial y \neq 0$).

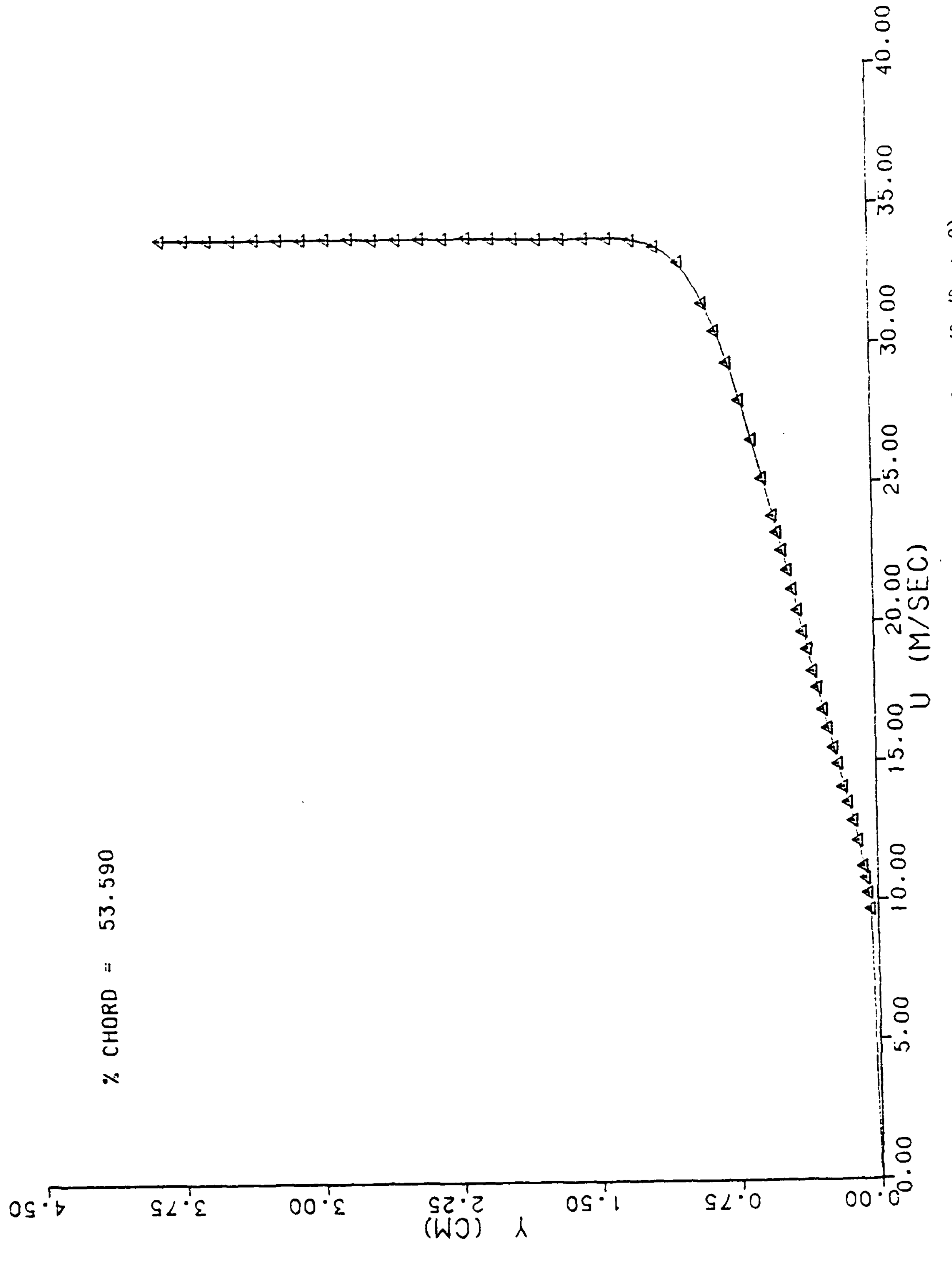


Figure 5. Spline fit of the boundary layer velocity profile ($\partial p/\partial y > 0$).

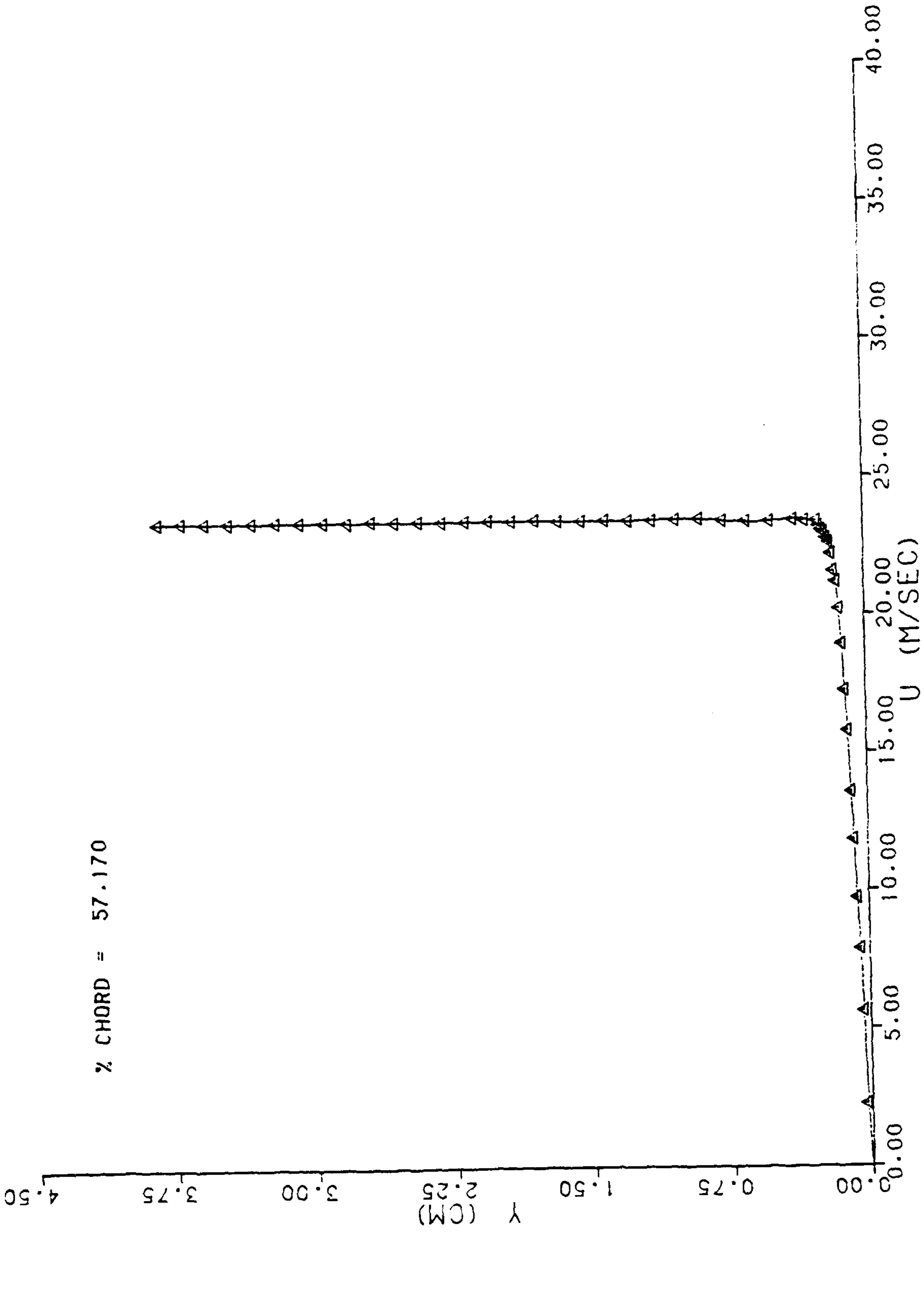


Figure 6. Spline fit of the boundary layer velocity profile ($\partial p/\partial y < 0$).

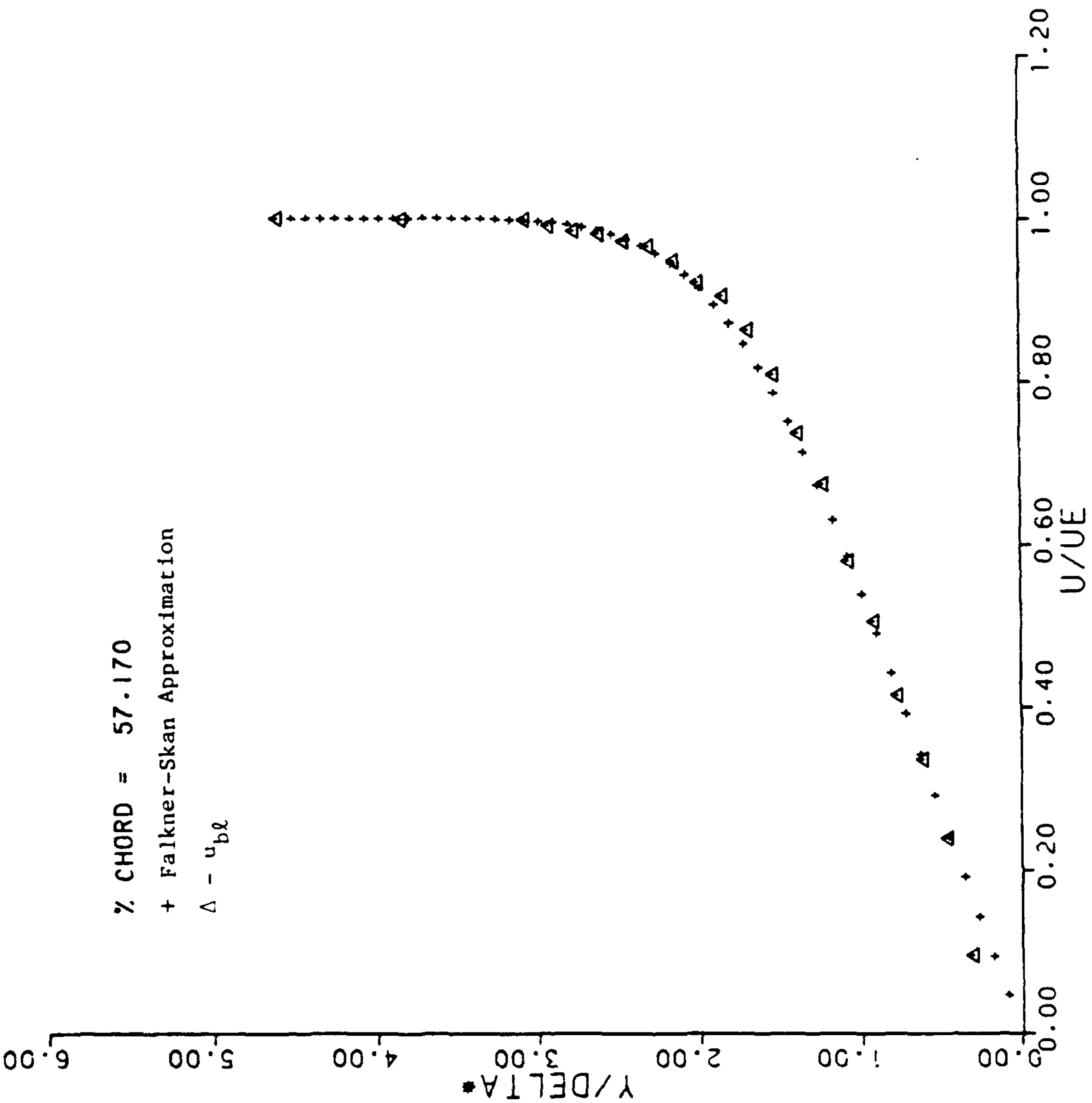


Figure 7. Falkner-Skan approximation to the boundary layer velocity profile ($\partial p/\partial y < 0$).

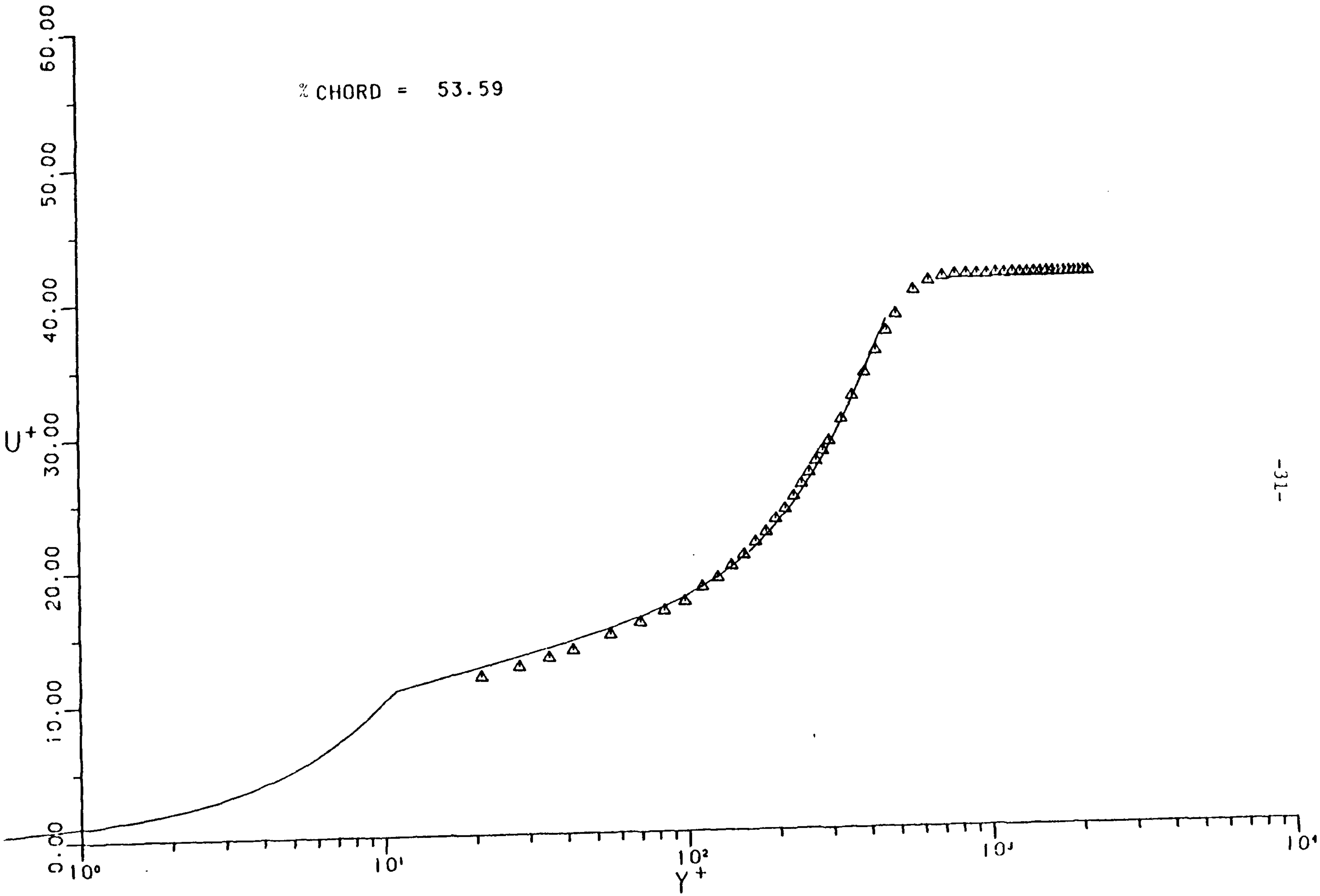
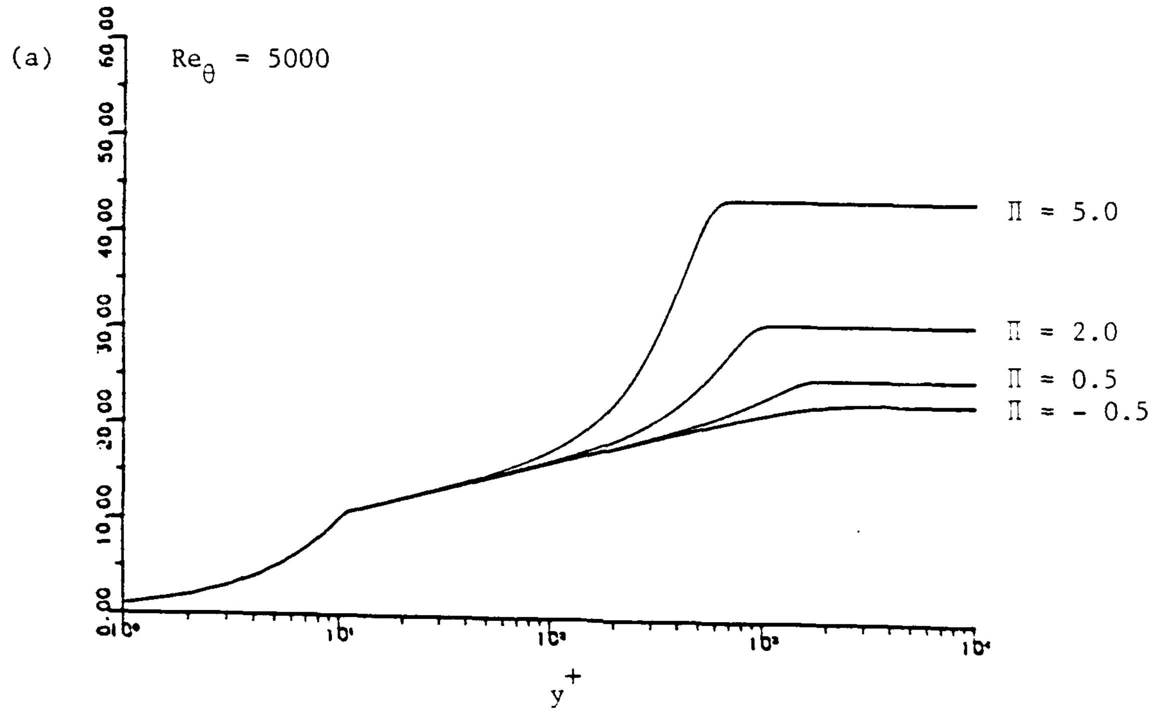


Figure 8. Wall/wake approximation to the boundary layer velocity profile ($\partial p/\partial y > 0$).



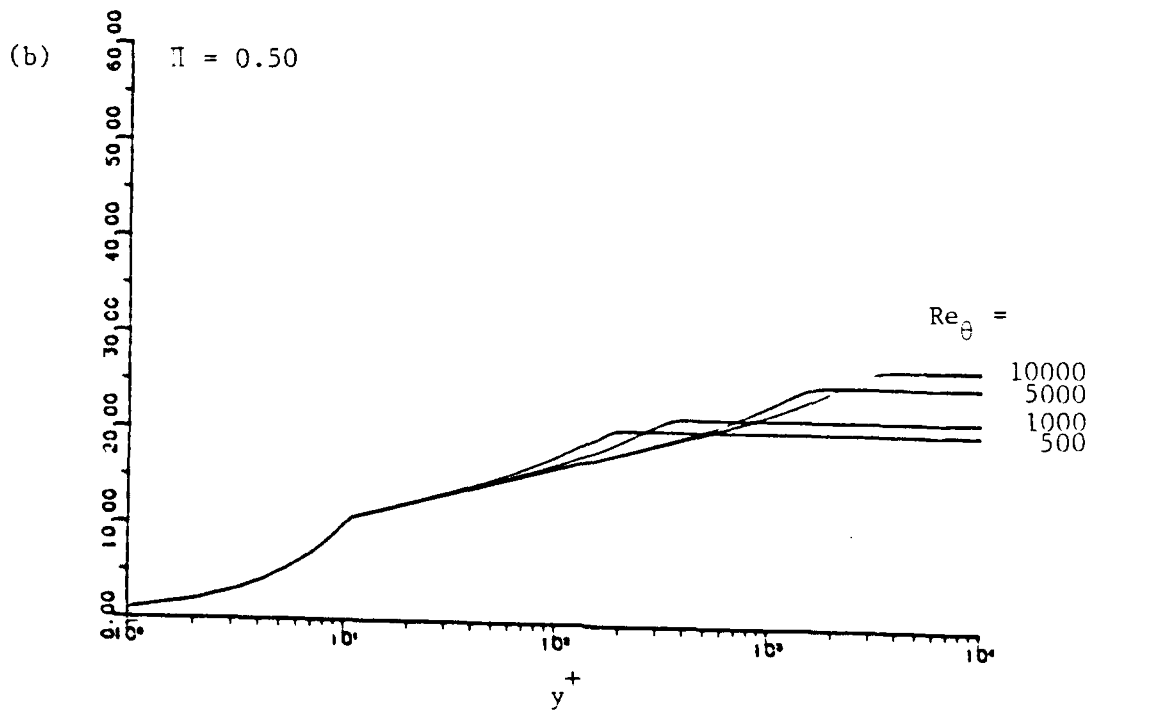


Figure 9. (a) Wall/wake composite profile: Π varies.
(b) Wall/wake composite profile: Re_θ varies.

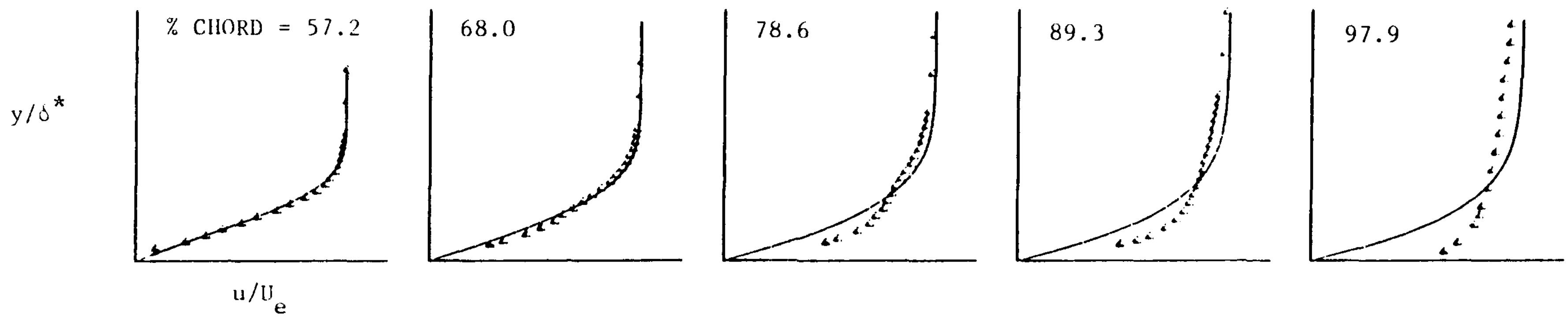


Figure 10. Incomplete transition of the pressure surface data: solid line is the Falkner-Skan approximation and the triangles represent the reconstructed boundary layer velocity profiles.

DISTRIBUTION LIST FOR UNCLASSIFIED TECHNICAL MEMORANDUM 85-85,
by W. C. Zierke and S. Deutsch, dated 25 May 1985

Director
Applied Physics Laboratory
The Johns Hopkins University
Johns Hopkins Road
Laurel, MD 20810
Attn: S. A. Gearhart
(Copy No. 1)

Commander
David W. Taylor Naval Ship
Research & Development Ctr.
Department of the Navy
Bethesda, MD 20084
Attn: J. H. McCarthy
Code 154
(Copy No. 2)

Commander
David W. Taylor Naval Ship
Research & Development Ctr.
Department of the Navy
Bethesda, MD 20084
Attn: F. B. Peterson
Code 1544
(Copy No. 3)

Commander
David W. Taylor Naval Ship
Research & Development Ctr.
Department of the Navy
Bethesda, MD 20084
Attn: M. M. Sevik
Code 19
(Copy No. 4)

NASA Lewis Research Ctr.
21000 Brookpark Road
Cleveland, OH 44135
Attn: C. Ball
(Copy No. 5)

NASA Lewis Research Ctr.
21000 Brookpark Road
Cleveland, OH 44135
Attn: D. Bolton
(Copy No. 6)

NASA Lewis Research Ctr.
21000 Brookpark Road
Cleveland, OH 44135
Attn: L. A. Povinelli
(Copy No. 7)

NASA Lewis Research Ctr.
21000 Brookpark Road
Cleveland, OH 44135
Attn: N. Sanger
(Copies 8 through 12)

NASA Lewis Research Ctr.
21000 Brookpark Road
Cleveland, OH 44135
Attn: T. Strazisar
(Copy No. 13)

NASA Lewis Research Ctr.
21000 Brookpark Road
Cleveland, OH 44135
Attn: J. R. Wood
(Copy No. 14)

National Bureau of Standards
Aerodynamics Sections
Washington, DC 20234
Attn: P. S. Klebanoff
(Copy No. 15)

Naval Research Laboratory
Department of the Navy
Washington, DC 20390
Attn: R. J. Hansen
(Copy No. 16)

Commander
Naval Sea Systems Command
Department of the Navy
Washington, DC 20362
Attn: Library
Code NSEA-09G32
(Copies 17 and 18)

DISTRIBUTION LIST FOR UNCLASSIFIED TECHNICAL MEMORANDUM 85-85,
by W. C. Zierke and S. Deutsch, dated 25 May 1985
[continuation]

Commander
Naval Sea Systems Command
Department of the Navy
Washington, DC 20362
Attn: T. E. Peirce
Code NSEA 63R-31
(Copy No. 19)

Commanding Officer
Naval Underwater Systems Ctr.
Department of the Navy
Newport, RI 02840
Attn: Library
Code 54
(Copy No. 20)

Office of Naval Research
800 North Quincy Street
Department of the Navy
Arlington, VA 22217
Attn: C. M. Lee
Code 432
(Copy No. 21)

Office of Naval Research
800 North Quincy Street
Department of the Navy
Arlington, VA 22217
Attn: M. M. Reischman
Code 432F
(Copy No. 22)

Office of Naval Research
800 North Quincy Street
Department of the Navy
Arlington, VA 22217
Attn: R. Whitehead
Code 432
(Copy No. 23)

Professor J. L. Lumley
Sibley School of Engineering
Cornell University
Ithaca, NY 14850
(Copy No. 24)

Dr. C. L. Merkle
Department of Mechanical
Engineering
The Pennsylvania State University
University Park, PA 16802
(Copy No. 25)

Professor T. H. Okiishi
Department of Mechanical
Engineering
Iowa State University
Ames, Iowa 50010
(Copy No. 26)

Professor W. M. Phillips
Chairman
Department of Mechanical
Engineering
Purdue University
Lafayette, IN 47907
(Copy No. 27)

Director
Applied Research Laboratory
The Pennsylvania State University
Post Office Box 30
State College, PA 16804
Attn: T. F. Davis
(Copy No. 28)

Director
Applied Research Laboratory
The Pennsylvania State University
Post Office Box 30
State College, PA 16804
Attn: S. Deutsch
(Copy No. 29)

Director
Applied Research Laboratory
The Pennsylvania State University
Post Office Box 30
State College, PA 16804
Attn: W. S. Gearhart
(Copy No. 30)

DISTRIBUTION LIST FOR UNCLASSIFIED TECHNICAL MEMORANDUM 85-85,
by W. C. Zierke and S. Deutsch, dated 25 May 1985
[continuation]

Director
Applied Research Laboratory
The Pennsylvania State University
Post Office Box 30
State College, PA 16804
Attn: R. E. Henderson
(Copy No. 31)

Director
Applied Research Laboratory
The Pennsylvania State University
Post Office Box 30
State College, PA 16804
Attn: B. R. Parkin
(Copy No. 32)

Director
Applied Research Laboratory
The Pennsylvania State University
Post Office Box 30
State College, PA 16804
Attn: W. C. Zierke
(Copy No. 33)

Director
Applied Research Laboratory
The Pennsylvania State University
Post Office Box 30
State College, PA 16804
Attn: GTWT Files
(Copy No. 34)

Director
Applied Research Laboratory
The Pennsylvania State University
Post Office Box 30
State College, PA 16804
Attn: ARL/PSU Library
(Copy No. 35)